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A: Files for Composite Trapezoid and Simpson’s Rule that main file will call

File For Composite Trapezoid Rule, Trap.m

function IntT=Trap(f,a,b,Ntot);

h = (b-a)/Ntot;

x = zeros(Ntot+1,1);

for i = 1:Ntot+1

x(i) = a + (i-1)\*h;

end

s = (f(x(1))+f(x(Ntot+1)))/2;

for i = 2:Ntot

s = s + f(x(i));

end

IntT = s\*h;

File For Composite Simpson’s Rule, Simp.m

function IntS=Simp(f,a,b,Ntot);

h = (b-a)/(Ntot);

for i = 1:Ntot+1

x(i) = a + (i-1)\*h;

end

s = f(a)+f(b);

for i = 1:(Ntot)/2

s = s + 2\*f(x(2\*i+1));

s = s + 4\*f(x(2\*i));

end

s = s - 2\*f(b);

s = s\*h/3;

IntS=s;

B and C

%Main file for integration

clear all

f = @(y) sqrt(1+y^2);

exact = (5/2)\*sqrt(26)-(1/2)\*log(-5+sqrt(26));

a = 0;

b = 5;

%f=@(y) 2+cos(y)\*cos(3\*y);

%exact=2\*pi

%a=0;

%b=2\*pi;

Nvec=[10 20 40 80 160];

n=length(Nvec);

for i=1:n

Ntot=Nvec(i);

IntT=Trap(f,a,b,Ntot);

ErT(i)=abs(exact-IntT);

h = (b-a)/Ntot;

ErTh(i)=ErT(i)/(h^2);

IntS=Simp(f,a,b,Ntot);

ErS(i)=abs(exact-IntS);

h = (b-a)/Ntot;

ErSh(i)=ErS(i)/(h^4);

end

Calculating the Error for

Diagram

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Table of Absolute Error

Table

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Simpson’s rule starts with lower error and keeps on decreasing since it is O(h^2) whereas Trapezoidal rule is O(h^2)—decreases but starts with higher error (10^-2 vs 10^-4 for Ntot=10). For trapezoid, as we double the # of intervals, we can see that the error decreases by about ¼ (e.g. Ntot=10 has ~.02 error and Ntot=20 has ~.005 error). This makes sense since Error ~ O(h^2) and doubling the # of intervals Ntot is equivalent to halving h, so Error was ~h^2 for Ntot=10 and then for Ntot=20, error should be ~(h/2)^2=h^2/4, so error should decrease by a factor of 4. Similarly, Simpson’s method is O(h^4), so halving h (doubling Ntot) should result in error decreasing by a factor of 2^4=16 and we see it is a bit off from N=10 to N=20, but fairly close to a factor of 16 for N=20 to 40 etc.

Table of Absolute Error / h^2 for Trapezoid and Absolute Error / h^4 for Simpson’s

Table

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Since the error is O(h^2), if we divide the error by h^2, we can start to find out what the constant in front of h^2 is. For Trapezoidal rule, we can see that it is ~.08. Again, for Simpson’s rule, the trend in the error is not as evident for the first two values of Ntot, but as we increase Ntot and decrease h, we see that the constant is ~.000024.

For the second integral with integrand 2+cos(x)sin(3x) on 0 to pi, this is the resulting table of absolute errors:

Table

Description automatically generated

The table above has approximately 0 error to start with this. This means that the theoretical error actually is canceling and we do not have any error. However, as we start to increase the # of intervals, Ntot, we actually see error starting to increase and this is due to the fact that the computations have round-off error.

And this is the resulting table when dividing by h^2 for Trapezoid and h^4 for Simpson’s.

Table

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Here, it is also hard to see the trend since error is so low.